Topology Change and Nonperturbative Instability of Black Holes in Quantum Gravity

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Abstract

Topology change in quantum gravity is considered. An exact wave function of the Universe is calculated for topological Chern-Simons 2+1 dimensional gravity. This wave function occurs as the effect of a quantum anomaly which leads to the induced gravity. We find that the wave function depends universally on the topology of the two-dimensional space. Indeed, the property of the ground state wave function of Chern-Simons gravity which has an attractive physical interpretation is that it becomes large in the infrared (large distances) if the Universe has "classical" topology $S^2 \times R$. On the other hand, nonclassical topologies $\Sigma_g \times R$, where Σ_g is the Riemann surface of genus g, are driven by quantum effects into the Planckian regime ("space-time foam"). The similar behavior of the quantum gravitational measure on four-manifolds constructed recently is discussed as the next example. We discuss the new phenomenon of the nonperturbative instability of black holes discovered recently. One finds that the Planck- sized black holes are unstable due to topology change. The decay rate is estimated using the instanton approximation. A possible solution to the primordial black hole problem in quantum cosmology is suggested.

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The topology of space or spacetime may play a fundamental role in any reasonable theory of quantum gravity [2-6]. The fascination with the possibility of topology change has led to several recent works [4,6,7,8,12-16]. It has recently been proposed that quantum gravity at ultra-short distances may be an example of a theory in which the interaction is introduced by a *topological principle* [13]. Indeed, splitting of Universes, or topology change can be thought of as a way to introduce interactions between extended objects (Universes). Second-quantized field theory of Universes will have topological perturbative expansion with topology change (cobordisms) playing the role of "Feynman diagrams" of an ordinary field theory. String theory (or 1+1 dimensional induced gravity) is the simplest example of such a model [13,15]. In analogy to an interacting string theory a model of an interacting second-quantized geometry was proposed [13]. The plethora of different topologies of low-dimensional manifolds for which the oriented cobordism groups are trivial (D=2,3) (D+1 dimensional Universes) seems to suggest that an interacting geometry model may indeed be very rich and may lead to many surprizes. In this essay I will address three issues of topology change, demonstrating that one may expect more surprises to come in the near future.

First I will demonstrate how, in the 2+1 dimensional Chern-Simons gravity, an exact wave function of the Universe arises as an effect of a quantum anomaly. The Chern-Simons model is an example of the topological field theory in 2+1 dimensions introduced some time ago [14]. In this model the spacetime metric does not seem to play any role on a compact manifold without boundary. However, we will see that the metric reappears as a dynamical degree of freedom as an effect of a quantum gravitational anomaly induced by the presence of boundaries. This phenomenon has an attractive physical interpretation. Starting with a theory without a metric explicitly appearing in the action, we obtain an induced gravity due to the effect of a quantum anomaly. This anomaly is exactly calculable and as its effect one obtains the wave function of the 2-dimensional Universe

$$\Psi_0[h_{ij}, \Sigma] = \int \mathcal{D}A \exp(iI_{CS}[A]) = \exp(cI_L[h_{ij}]), \tag{1}$$

where $I_L[h]$ is the nonlocal Liouville action

$$I_L[h;\Sigma] = (96\pi)^{-1} \int\limits_{\Sigma} d^2x d^2y \sqrt{h(x)}\sqrt{h(y)} R(x) G(x,y) R(y) + ..., \qquad (2)$$

appearing frequently in string theory or two-dimensional induced gravity [17], c is a positive constant which is interpreted as a central charge in the conformal field theory, and $G(x,y)$ is the Green function of the conformally invariant scalar Laplacian. The central charge is $c = 6$ for the Chern-Simons gravity [16]. The reason the wave function Ψ_0 depends on a metric h_{ij} on the boundary Σ is the following. In order to evaluate the functional integral (1) one must fix a gauge $\nabla^a A_a = 0$. The gauge fixing procedure requires that one introduces a metric g_{ab} on a three-manifold $\mathcal{M}$ with a boundary Σ. One finds that the functional integral (1) for the Chern-Simons gravity is simply $\Psi_0 = \left(det'^{-1/4}\Delta_1 det'^{3/4}\Delta_0\right)^6$, where Δ_0 and Δ_1 are Laplacians on scalars and 1-forms respectively. In the case when a three-manifold is S^3 one finds that $\delta_g \Psi = 0$, i.e., Ψ_0 is independent of the metric g_{ab}. On the other hand on D^3 one finds a gravitational anomaly:

$$\delta ln\Psi_0 = \frac{c}{24\pi} \int\limits_{\Sigma} d^2x \sqrt{h} \delta\sigma (R + \mu^2), \qquad (1')$$

where $h_{ij} = e^{2\sigma}\gamma_{ij}$ is a metric on Σ, $c = 6$, and μ is a constant. The reason why a gravitational anomaly occurs is quite simple. The conformal anomaly in odd dimensions vanishes on closed manifolds but it is present if a manifold has a boundary!

One may then ask the question: what does this result mean? I will argue that once we are given the exact wave function of the Universe, the obvious thing to do is to find the appropriate Wheeler-De Witt equation for which $\Psi_0[h]$ is the solution. Indeed, one may look for the effective low-energy gravitational action $S[g]$ for which the Hartle-Hawking wave function of the Universe [9] coincides with the exact wave function $\Psi_0[h;\Sigma]$ of the induced gravity. We are led to consider the following Hartle-Hawking functional integral representation of $\Psi_0[h;\Sigma]$

: $\Psi_0[h;\Sigma] = \int_{h=g_{|\Sigma}} \mathcal{D}g e^{-S[g]}$. It is perhaps not surprising to find that, for small fluctuations δh around the boundary value $h = g_{|\Sigma}$ of the classical solution g to $\delta S = 0$, where $S[g] = \frac{1}{2\kappa}\int \sqrt{g} d^3x(R - \Lambda)$, the gaussian wave function is given by $\Psi_0[\delta h] = \exp\Big(a I^{(2)}{}_L[\delta h]\Big)$. $I^{(2)}{}_L[\delta h]$ is the gaussian part of the Liouville action and a is an undetermined constant. This result for Einstein-Hilbert 2+1 dimensional quantum cosmology was noticed some time ago but its physical meaning was not quite clear. The observation described above finds its interpretation and becomes a logical consequence of the gauge Chern-Simons model of gravity proposed recently by Witten [14]. Indeed, Witten has shown that classically the Einstein-Hilbert 2+1 dimensional gravity is equivalent to an exactly soluble Chern-Simons gravity which is the gauge theory of the 2+1 dimensional Poincare group. The action of the Chern-Simons gravity is

$$I_{CS}[e,\omega] = \frac{1}{4\pi}\int d^3x \epsilon_{\alpha\beta\gamma}\epsilon^{abc} e^{\alpha}{}_a(\partial_b \omega^{\beta\gamma}{}_c + \omega^{\beta\delta}{}_b \omega^{\delta\gamma}{}_c), \tag{3}$$

where $e^{\alpha}{}_a$ is a dreibein and $\omega^{\alpha\beta}{}_a$ is a $SO(2,1)$ spin connection. $A = (e,\omega)$ are the gauge fields corresponding to the 2+1 dimensional Poincare group. This example shows explicitly that two models which are equivalent classically may lead to qualitatively different quantum field theories. Indeed, Einstein-Hilbert gravity in 2+1 dimensions, first studied by Staruszkiewicz, leads to a nonrenormalizable QFT but the C-S gravity is renormalizable with a vanishing beta function of the renormalization group.

In string theory the conformal anomaly leads to the 1+1 dimensional induced gravity Liouville action I_L. On the other hand in the gauge Chern-Simons models this is the wave function of the Universe $\Psi_0[e^{2\sigma}h_{ij}] = \exp(cI_L[h;\sigma])$ which is generated by a quantum anomaly. As Witten pointed out these models are uniquely suited for the calculation of the topology changing amplitudes [14]. A similar observation was described by Nair and the present author [13]. In effect it was proposed that a topological Yang-Mills field theory may lead to induced gravity due to quantum anomaly. We suggested also that topological field theories are good models to study topology change in quantum gravity [13].

I will not address here the issue of calculation of topology changing amplitudes. Rather, I describe the physical implications of the Liouville wave function of the Universe in a induced gravity [16]. We observe that because there are no gravitons in 2+1 dimensional gravity only the conformal factor σ of the two-metric $h_{ij} = e^{2\sigma}\gamma_{ij}$, where γ_{ij} is a fixed metric of a constant curvature, enters the wave function $\Psi_0[h]$. What happens when the distances on Σ_g are enlarged by a constant rescaling, $L_0 \to L = e^{\sigma} L_0$? The wave function $\Psi_0[h]$ changes universally as

$$\Psi_0\left[(L/L_0)^2 h; \Sigma_g\right] = \left(\frac{L}{L_0}\right)^{\frac{c}{6}\chi} \Psi_0[h; \Sigma_g], \tag{4}$$

where χ is the Euler characteristic of Σ_g. What is the meaning of this anomalous scaling of the wave function? It is reminiscent of the finite size effects in statistical physics models in two dimensions. First of all, we observe that the dependence of the wave function on the Euler characteristic $\chi = 2(1-g)$ discriminates between different topologies. Indeed, if $\chi > 0$, $g = 0$, $\Sigma = S^2$ (or $\Sigma = D^2$) the wave function diverges for large size two-geometries. This infrared instability has an attractive physical interpretation. The quantum dynamics of two-geometries with the simple, "classical" topology of the two-sphere S^2 drives the Universe into the infrared, long distance classical regime. What we are witnessing here is the "birth" of a macroscopic Universe.

In the long-distance regime the effective action describing dynamics of gravity is the Einstein-Hilbert action. This is because in the long wavelength limit only terms with the lowest number of derivatives of a metric survive. One finds that, on the classical topology $S^2 \times R$, the low energy effective action leads to physically acceptable solutions. Indeed, there exists the singularity-free inflationary 2+1 dimensional De Sitter Universe solution. On the other hand, if one looks for classical solutions on $\Sigma_g \times R$ topologies one encounters singularities. The occurence of singularities in the $\Sigma_g \times R$ cosmological models has a simple physical interpretation. The wave function diverges at Planckian distances $L \leq L_0$, for Universes with topology Σ_g, $g \geq 2$. At short distances, the effective action $S[g_{ab}]$ which leads to

the wave function $\Psi_0[h; \Sigma_g]$ must start with terms containing an arbitrary number of derivatives of a metric g_{ab}. In other words, in the ultra-short Planck distance regime all derivatives of the metric are equally important. This means that the effective action $S[g_{ab}]$ is a highly nonlocal functional of the three-metric. Therefore it is not surprising that one simply cannot use the Einstein-Hilbert gravity to describe cosmological models with nontrivial topology. The occurence of singularities on nontrivial topologies has its simple explanation in the fact that such Universes never appear in the classical limit of large distances. They dominate the dynamics of the 2+1 dimensional QG at Planckian distances. Trying to enforce the classical dynamics on such Universes leads immediately to pathologies, i.e., singularities. This phenomenon may shed some light on the issue of singularities in the 3+1 dimensional gravity. This leads us to the second issue I would like to discuss in this essay.

The problem posed some time ago by Hawking [3] is how the "spacetime foam" picture [2] arises in quantum gravity. I would like to argue that the proper approach to this problem is the construction of the correct gravitational measure on the space of random four-geometries. In any theory of quantum gravity with the metric g_{ab} treated as the fundamental field variable, we must know the quantum measure on the space of metrics on a given four-manifold. The quantum theory is defined by the Feynman sum over histories. This formulation has two important ingredients. First, one has to specify the space of dynamical variables and construct the quantum measure on this space. The second step is to postulate the action principle on the space of "histories". Once this is done we have a formal definition of a quantum theory.

It happens frequently that the space of dynamical variables of some theory can be equiped with a Riemannian geometry. Examples of such theories are the quantum string and quantum gravity. The important idea due to Polyakov [17] is to put this Riemannian geometrical structure of the space of fields to work. Now it is a property of Riemannian geometry that a volume form is determined by a metric. One needs to specify an appropriate coordinate system on the manifold

and the "square-root of the determinant of the metric" rule then helps to find the measure. Changing a coordinate system on the manifold introduces Jacobian factors which are calculable. We calculate the measure on the space of deformations of four-geometries using the Polyakov method [17]. The basic motivation for the construction of the gravitational measure on an arbitrary topological four-manifold is to set up the formalism for the calculation of topology changing amplitudes in quantum gravity. In this essay we focuss attention on the case of manifolds without boundary. We find that the anomalous scaling of the quantum measure depends universally on the topology of $\mathcal{M}$.

Consider the point g_{ab} on the superspace Q, i.e., space of metrics, and the tangent space $TQ_{|g}$ at g. The tangent space to Q at g is spanned by the metric deformations δg_{ab}. Consider now the class of conformal and diffeomorphism deformations of the metric g_{ab}: $\delta g_{ab} = (2\delta\sigma + \frac{1}{2}\nabla^c\xi_c)g_{ab} + (L\xi)_{ab}$, where the operator L maps vectors (one-forms) into symmmetric traceless two-tensors $(L\xi)_{ab} = \nabla_a\xi_b + \nabla_b\xi_a - \frac{1}{2}g_{ab}\nabla^c\xi_c$. The operator L describes the traceless piece of the deformation induced by a diffeomorphism generated by a vector field ξ^a. Thus, the only deformations δg_{ab} which are not obtained by diffeomorphisms and conformal rescaling are in the complement of the Range$L$ in the tangent space $TQ_{|g}$. Now we would like to have an orthogonal decomposition of the tangent space at g, i.e., the orthogonal decomposition of δg_{ab}. In order to do that we need a metric on the space of deformations. The condition of ultralocality of the measure dictates the minimal choice of the "covariant" metric G^{abcd} on the space of deformations, so that $||\delta g_{ab}||^2 = \int_M d^4x g^{1/2} G^{abcd}\delta g_{ab}\delta g_{cd}$, where $G^{abcd} = \frac{1}{2}\left(g^{ac}g^{bd} + g^{ad}g^{bc} + Cg^{ab}g^{cd}\right)$, and C is an arbitrary constant for which the norm is positive definite. The decomposition which is orthogonal with respect to this metric is: $\delta g_{ab} = 2\delta\sigma + \mathrm{Range}L + (\mathrm{Range}L)^T$. Now we know that the $(\mathrm{Range}L)^T$ is the same as $\mathrm{Ker}L^\dagger$, where $L^\dagger$ is the adjoint of L with respect to the scalar product $<,>$: $(L^\dagger h)_a = -2\nabla^b h_{ab}$.

The Range$L$ part of this decomposition can be gauged away by $\mathrm{Diff}_0(\mathcal{M})$. This part of the deformation will contribute in the measure an infinite volume of the connected to identity diffemorphism group $\mathrm{Diff}_0(\mathcal{M})$. Therefore it can be

factored out from the measure $\mathcal{D}\delta g_{ab}$. The defining condition for the measure is: $\int \mathcal{D}\delta g \exp\Big(-\frac{1}{2} < \delta g, \delta g >\Big) = 1$. Using the natural orthogonal decomposition of the tangent space to Q we find

$$d\mu(g) = \mathcal{D}\delta g = J\mathcal{D}\sigma\mathcal{D}\xi\mathcal{D}h, \tag{5}$$

where the Jacobian J is $J = det'^{1/2}L^{\dagger}L$. The measure $d\mu(g)$ depends on the point g on the space of metrics Q [15]. If we rescale the metric g by a constant $e^{2\sigma} = \Big(\frac{\lambda}{\lambda_0}\Big)^2$ then the measure will respond to that rescaling: $\frac{d}{d\sigma} lnJ = c\chi$, where χ is the Euler number of $\mathcal{M}$. We find that the anomalous scaling of the measure is: $d\mu\Big((\frac{\lambda}{\lambda_0})^2 g\Big) = \Big(\frac{\lambda}{\lambda_0}\Big)^{c\chi} d\mu(g)$, where $c = \frac{257}{288}$.

The details of the derivation of this result are technically involved and are presented in a recent paper [16]. The anomalous scaling of the measure depends universally on the topology of $\mathcal{M}$. We find it very attractive indeed, that the measure diverges in the infrared regime $\lambda \to \infty$ for topologies with a positive Euler number $\chi > 0$. An example of a manifold admitting the classical Einstein metric is S^4 (the Euclidean De Sitter which plays such an important role in the Hawking-Coleman resolution of the cosmological constant problem). Indeed, a manifold admitting an Einstein metric with $\Lambda > 0$ must have positive Euler number. Another example of an important solution with a positive Euler number ($\chi = 2$) is the Euclidean Schwarzschild black hole. We will discuss later a new phenomenon of nonperturbative instability of black holes by topology change [12].

The quantum gravitational measure favors large Universes with "classical" topologies. This infrared instability has the same interpretation as before. We are witnessing the "birth" of a macroscopic Universe. The quantum gravitational dynamics seems to drive "classical" geometries to the infrared regime. On the other hand the "nonclassical" topologies, like a sphere S^4 with a number h of "handles" $S^3 \times S^1$, $\chi = 2(1 - h)$, will be driven by quantum dynamics to the Planckian regime! In other words topologies with $\chi < 0$ will dominate the quantum gravitational vacuum. They are the elements ("building blocks") of "spacetime foam".

Thus, the exact quantum measure leads to a clear cut demonstration of the "space-time foam" picture of Wheeler and Hawking [2,3]. For topologies with a negative Euler number one cannot expect that the Einstein-Hilbert action will adequately describe the effective gravitational dynamics. Indeed, one expects the occurence of singularities and causality violation on such topologies. The issue of singularities which occur in the gravitational collapse is intimately connected to the problem of the final stages of a black hole evaporation due to the Hawking effect [1].

I will demonstrate that once the effects of quantum gravity are included a black hole does not evaporate completely losing its energy steadily to a flux of created particles, but rather decays via a change in topology into an asymptotically flat space and an object which is a closed Friedmann Universe [12]. This process is a genuine nonperturbative effect of quantum gravity and becomes the dominant "channel" of a black hole decay for black holes with masses slightly larger than the Planck mass $M_p = 10^{19}$GeV. The nontrivial topology of black holes and their highly nontrivial quantum mechanical properties is probably the strongest argument in favor of considering quantum fluctuations in the topology of spacetime. Some time ago Zel'dovich suggested that small black holes with masses close to the Planck mass M_p would decay in one quantum jump and a small closed world will be formed in such a process [5] together with an asymptotically Minkowski spacetime. Zel'dovich envisaged that such a process would necessarily violate the baryon and lepton number conservation law [5].

Indeed, a very small black hole of mass M comparable to the Planck mass M_p has a very high Hawking temperature $T = {M_p}^2/8\pi M$. The average thermal energy of the particles emitted by such black holes is slightly below the Planck mass, and it is definitely favorable for a black hole to "disappear" in a quantum fluctuation (as a discrete "quantum jump"), with a possible change of topology [5]. Also the average number N of particles produced in the black hole decay is of order $N \cong 8\pi M^2/{M_p}^2$. Notice that this number is proportional to the geometrical scattering cross-section or the area of a black hole horizon. Imagine now that a black hole is formed as an intermediate state in the collision of high energy elementary particles, say baryons.

Such a state would decay rapidly producing a number of other particles. The metastable intermediate state tends to behave thermodynamically as the number of "fragments" increases, which indicates that, even if the quantum coherence is not lost in such a process, the phase space becomes very large. In the fundamental theory of all interactions quantum black holes would probably appear as such "resonances", or collective excitations, which eventually would "fragment" into a number of particles. The fact that the quantum mechanical decay of such a state can be described by thermodynamics simply reflects the hierarchy problem in quantum gravity (QG). Indeed, the mass scale of QG M_p is much above the energy scale of other fundamental interactions. It seems that this property of QG might be responsible for the large number of particles produced in the decay of a quantum black hole. The standard picture of final stages of the black hole evaporation does not seem to take into account the quantum effects of gravity.

We can presumably use the Wheeler-De Witt (WDW) equation for the description of small "quantum black holes". Consider the real time, i.e., Lorentzian, configuration of the gravitational field, say a black hole. In the quantum mechanics of the gravitational field, one associates a wave function to this configuration. It is known how to do this, at least semiclassically, in a WKB approximation. One considers a Riemannian three-manifold Σ which is an initial data hypersurface Σ in classical general relativity (GR). The initial data is a canonical pair (h_{ij}, π_{ij}), a "point" in the phase space where the semiclassical wave function is localized. Usually one chooses the polarization on the classical phase space such that the WDW wave function is a functional of the three-metric $h_{ij} : \Psi[h_{ij}]$.

A "ground state" wave function may cease to be gaussian in some directions in the configuration space. This behavior signals instability. The wave function tunnels to a classically forbidden region. Such a tunnelling is most conveniently described in the path integral approach where one can calculate the transition amplitude in a WKB approximation. This leads automatically to an instanton mediating such a transition.

In general relativity a black hole is stable. The topology of Σ does not change in classical general relativity because it would lead to causality violation. Therefore, the system under consideration tunnels to another classically allowed region of the phase space if a semiclassical instability is really present. In general relativity (GR) the tunnelling can be associated with topology change [8,15].

Studying the semiclassical instability of black holes due to topology change in QG, one must find an instanton which mediates this instability. The classical "ground state" of a black hole is uniquely described by the Schwarzschild solution, which has the topology $R^2 \times S^2$. The presence of an instability can be established most easily by analytically continuing this solution to a Euclidean space signature, so that the metric is

$$ds^2 = (1 - 2GMr^{-1})d\tau^2 + (1 - 2GMr^{-1})^{-1} + r^2 d\Omega^2, \qquad (6)$$

where M is a black hole mass and $G = {M_p}^{-2}$ is the Newton constant given in terms of the Planck mass M_p. The Schwarzschild radius is $R = 2M{M_p}^{-2}$. The constant time section of this geometry has topology $R \times S^2$. This is the famous Einstein-Rosen bridge or the three-dimensional wormhole connecting two asymtotically flat regions. If one restricted the range of the radial coordinate to $R < r < \infty$, this three-geometry would be an incomplete manifold. However, one can see that the origin of this incompleteness is the presence of a "hole" at $r = R$. Physically the initial constant time section is only half of the Einstein-Rosen bridge. One can find a coordinate system on half of the wormhole such that the metric is conformal to the metric on half of the three-sphere S^3.

How do we search for a semiclassical instability of some given configuration? The proper tool we need to address the question of topology change is cobordism theory [8]. Let Σ be an initial three-geometry and Σ' be a final three-geometry. The question of semiclassical instability of Σ can be formulated now as the problem of the existence of a smooth Riemannian manifold $\mathcal{M}$ interpolating between Σ and Σ'. Two oriented manifolds are called cobordant if their disjoint union bounds

a smooth manifold, $\partial\mathcal{M} = \Sigma \cup \Sigma'$. The basic result of cobordism theory which is useful here is that all closed oriented three-manifolds are cobordant. This also means that S^3 can "decay" into any closed arbitrarily complicated oriented three-manifold. If a manifold is compact and has a boundary then one can modify the present argument and consider cobordisms with fixed boundaries.

Now a three-sphere with a boundary S^2 is cobordant to any oriented three-manifold with S^2 boundary. This means that, at least in principle, a black hole can decay into any topologically nontrivial configuration. Consider the complete three-manifold obtained by "filling in" a hole in R^3, i.e., by gluing in a disk D^3 (or S^3 with a boundary S^2) to half of the Einstein-Rosen bridge: $R \times S^2 + D^3 = R^3$. Adding a sphere S^3 to a disk D^3 ($D^3 \cup_g S^3 \equiv D^3$) does not change topology of a disk: $R \times S^2 + D^3 \cup_g S^3 = R^3 + S^3$, but rather corresponds to a process of producing a disjoint union of R^3 and S^3. This "cupping" operation produces a complete manifold R^3 (or $R^3 + S^3$). The procedure described above is performed on the constant time initial data three-geometry. It corresponds to the "three geometry" interaction "vertex", i.e., the process $BH \to AF + CW$ (here AF and CW denote the asymptotically flat and closed world spacetimes). This "three geometry" interaction must be described by the second-quantized interacting geometry model of quantum gravity [13]. One may expect that the semiclassical approach described below captures the essential qualitative properties of the decay mechanism. One would like to find an instanton solution corresponding to this decay mode. The simplest possible way to obtain such an instanton is to match a Euclidean black hole solution to a Friedmann universe on the constant time hypersurface. However, there are no vacuum solutions corresponding to this "match". Consider a four manifold $\mathcal{M}_F$ of topology $S^3 \times R$ with a minimal S^3. Cut a wormhole $\mathcal{M}_F$ in half on the minimal S^3. Next cut this minimal S^3, a constant "time" slice of a wormhole geometry, along an equator S^2 obtaining thereby a disk D^3. A disk D^3 is topologically the same as a disk D^3 and a sphere S^3 glued together: $D^3 \equiv D^3 \cup_g S^3$. Now glue the constant time slice of the Euclidean Schwarzschild spacetime (ES) along the "horizon" S^2 to an equator of the minimal S^3 of the half-wormhole. This

operation defines the manifold of an instanton which mediates the decay of a black hole to a closed Friedmann universe and an asymptotically flat spacetime without a hole. The hybrid four-manifold obtained this way is the ES on the one side of the hypersurface of a constant time and the Friedmann universe on the other side. It specifies also the initial data for the AF space to which a black hole decays. One may argue that the details of the AF space are not relevant because the only thing of interest is the decay rate of a black hole decaying to "anything".

Now we have to find a solution to the Euclidean equations of motion corresponding to the process of "pinching off" a small closed universe. Indeed, one can find such an instanton solution for the axion two-form B coupled to gravity. Effects of the axion B field on black holes in string theory were first considered in Ref. 7. We simply observe that the Bekenstein result [11] which says that a static black hole does not have scalar (spin 0) hair can be easily extended to the Euclidean instanton case. The instanton solution in question is simply the ES on one part of an instanton manifold and a four-dimensional wormhole on the other part of a complete instanton manifold. One simply requires smooth matching conditions on the metric and the second fundamental form of a matching surface. Indeed, one can show that such a smooth matching of the ES and the Friedmann universe does exist (note that the matching is achieved on the constant time hypersurfaces).

The Euclidean Schwarzschild solution is known to possess the amazing property of having one normalizable negative mode in the "graviton" sector. What does this mean physically? Gross et al. [10] interpreted this fact as corresponding to a semiclassical nonperturbative instability of flat Minkowski spacetime in a thermal bath due to the nucleation of black holes. Mathematically, this means that the ES instanton is only a local extremum of the Euclidean Einstein-Hilbert action.

The four-dimensional wormhole simply has the Euclidean Friedmann metric which is the solution of the coupled axion and Einstein equations. One can show that this solution is absolutely stable. In the linear approximation, one finds no negative modes in the spectrum of deformations of the wormhole. On the other

hand the half-wormhole does have negative modes in its spectrum of deformations.

The instanton solution described above mediates the decay of a black hole into a small closed universe "pinching off" from our "large" Universe containing a black hole. There exist negative modes in the spectrum of fluctuations around this instanton. Indeed, a wormhole cut in half and the Euclidean Schwarzschild do have negative modes. Therefore, the "matched" instanton solution corresponding to topology change $S^2 \times R + D^3$ to $S^3 + R^3$, and from $R^2 \times S^2$ to $R \times S^3 + R^4$, has negative modes in the "graviton" sector. In fact, there is only one normalizable negative mode around this instanton [15].

It is sufficient to present here the asymptotic form (as $\tau \to +\infty$) of the metric on the wormhole cut in half $ds^2 = d\tau^2 + a^2(\tau) d{\Omega_3}^2$, $a^2(\tau) \cong \tau^2(1 + 2R^4/3\tau^4)$. In a different form this solution was derived in Ref.18. Indeed, one shows that the exact solution to the coupled Einstein and the axion B-field equations can be given in the parametric form: $a(\eta) = R(cosh2\eta)^{1/2}$, $\tau = \int d\eta a(\eta)$, and $dB = g(\tau)\epsilon$, where ϵ is the volume 3-form on S^3. From the axion equations of motion: $d * H = 0$, $H = dB$, one finds $g = q/2\pi^2 f^2 a^3(\tau)$, where q is an integer global axion charge. Here $d{\Omega_3}^2$ is the metric on a unit round sphere S^3 and R is the radius of the minimal S^3 which equals the Schwarzschild radius. Indeed, this radius R is determined by the condition that the Euclidean Friedmann solution matches the Euclidean Schwarzschild solution. R depends on the axion coupling f and the global charge q. One finds the Euclidean action of the wormhole: $I_W = 0$, whereas the action of the ES instanton is: $I_S = \pi {M_p}^2 R^2$. The total action of the instanton is: $I = I_S + I_W = \pi {M_p}^2 R^2$. In terms of the black hole mass, the action is $I = 4\pi M^2/{M_p}^2$, where we have used $R = 2M{M_p}^{-2}$.

We find the decay rate of a black hole: $\Gamma \cong O(1){M_p}^5 M^{-1} \exp(-4\pi M^2/{M_p}^2)$. For a large black hole the decay rate due to this nonperturbative instability is extremely small and the decay time is much larger than the age of the Universe. However, for the mass of a small black hole, $M = M_p$ the decay rate per unit spacetime volume is only of order 10^{-6}. The spontaneous decay rate might indeed

be small but the decay stimulated by the environment might be much higher. Indeed, this is what we may expect to happen in the very early post-Planckian Universe. The mechanism described in this essay might explain the absence of primordial black holes. Indeed, it seems that unlike the monopole problem the primordial black hole problem is easier to resolve because black holes are unstable, while monopoles are stable.

It should be noticed that the decay rate of a black hole due to the nonperturbative instability is proportional to $\exp(-S_{bh})$, where S_{bh} is the Bekenstein-Hawking entropy. Indeed, the naive estimate of a number of microstates which correspond to a given black hole of the mass M is $N \cong \Gamma^{-1}$. The microcanical entropy for such a state is: $S_{bh} = lnN \cong -ln\Gamma = 4\pi M^2 {M_p}^{-2}$.

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